Chapter 11

Linear Equations in Two Unknowns

Learning Objectives

After completing this chapter, you will be able to

- understand linear equations in two unknowns.
- plot the graphs of linear equations in two unknowns.
- use linear equations in two unknowns and their graphs to solve some practical problems.

The cost for printing a comic book includes a basic fixed cost and a variable cost based on the number of copies printed. It costs \$800 for printing 50 copies and \$2 000 for 200 copies. How much does it cost for printing 100 copies?

Preview

[Basic knowledge and techniques required for this chapter.]

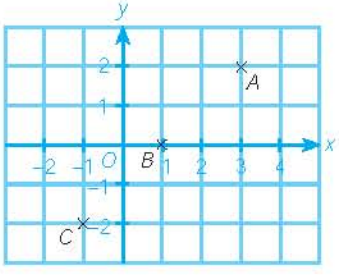
A. Basic Knowledge

1. Using algebraic expressions to represent statements

Statement	Algebraic expression	Variable	Number of variables
Total amount saved by Mick after n days if he has saved \$1 and starts saving \$2 every day	$\$(2n + 1)$	n	1
The total price of 1 dozen of pencils and $\frac{1}{2}$ dozen of notebooks if the selling prices of each pencil and notebook are \$$x$ and \$$y$ respectively	$\$(12x + 6y)$	x and y	2

2. Locating the position of a point on a coordinate plane

Point	A	B	C
Coordinates	(3, 2)	(1, 0)	(–1, –2)



B. Basic Technique

1. Substitution

Example: Find the value of the algebraic expression $3x - 4y + 2$ when

(a) $x = 1$, $y = 3$.

(b) $x = -2$, $y = 5$.

Solution: **(a)** When $x = 1$ and $y = 3$,

$$3x - 4y + 2 = 3(1) - 4(3) + 2 = \underline{\underline{-7}}$$

(b) When $x = -2$ and $y = 5$,

$$3x - 4y + 2 = 3(-2) - 4(5) + 2 = \underline{\underline{-24}}$$

2. Solving linear equation in one unknown

Example: Solve the equation $\frac{3x - 4}{2} = 7$.

Solution:
$$\frac{3x - 4}{2} = 7$$
$$3x - 4 = 14$$
$$3x = 18$$
$$x = \frac{18}{3}$$
$$= \underline{\underline{6}}$$

11.1 Linear Equations in Two Unknowns

Class Activity 11.1

Aim: To find out the possibility of expressing some practical problems as linear equations in two unknowns

Show Ans.

Single cable car fares for an adult and a child are \$50 and \$30 respectively. A group of tourists have spent \$410 on the fares.

(a) Let x and y be the number of adults and children respectively.

$\therefore$ Total fares for adults = \$_____

Total fares for children = \$_____

(b) Set up a linear equation according to the total fares of \$410.

_____

(c) In the above equation, there are _____ unknowns, they are _____ and _____.

Moreover, the degree of each unknown is _____.

(d) Can you find some solutions for the equation in **(b)**? (Solutions are values that can balance both sides of an equation after they are substituted into the unknowns.)

_____

(e) In the above situation, how many adults and children are there? (There can be more than one set of solutions.)

_____

Now I see ...

There are some linear equations with two unknowns, and these equations can have more than one set of solutions.

If an equation has two unknowns, and the highest degree of all terms is 1, this equation is called a **linear equation in two unknowns**.

The following are some linear equations in two unknowns x and y.

$50x + 30y = 410$, $y = 2x + 1$, $2x = 5y - 3$

linear equation in two unknowns 二元一次方程

Solutions are values which can balance both sides of an equation after they are substituted into the unknowns. Generally, a linear equation in two unknowns can have more than one set of solutions.

e.g. **(a)** Consider the equation $50x + 30y = 410$.
The following are some solutions:
$x = 1, y = 12$; $x = 4, y = 7$; $x = 7, y = 2$
which can also be expressed in ordered pairs:
(1, 12), (4, 7), (7, 2)

(b) Consider the equation $y = 2x + 1$.
The following are some solutions:
(0, 1), (1, 3), (1.5, 4), (−2.5, −4)

Additional Example 11.1

Example 11.1 Finding solutions of a linear equation in two unknowns

According to the following table, find four sets of the solutions of the equation $x + 2y - 5 = 0$, give your answers in ordered pairs.

x	−3	2		
y			1	0

Solution

[Analysis: After substituting a known value into the corresponding unknown, the equation becomes a linear equation in one unknown, and hence the value of another unknown can be found.]

For the equation $x + 2y - 5 = 0$,
substitute $x = -3$ into the equation,
$-3 + 2y - 5 = 0$
$2y = 8$
$y = 4$
$\therefore$ (−3, 4) is a solution.

Substitute $x = 2$ into the equation,
$2 + 2y - 5 = 0$
$2y = 3$
$y = 1.5$
$\therefore$ (2, 1.5) is a solution.

Substitute $y = 1$ into the equation,
$x + 2(1) - 5 = 0$
$x = 3$
$\therefore$ (3, 1) is a solution.

Classwork 11.1

Solution

According to the following table, find four sets of the solutions of the equation $2x - y = 6$, give your answers in ordered pairs.

x	−2		3	
y		−7		7

Substitute $y = 0$ into the equation,

$x + 2(0) - 5 = 0$

$x = 5$

$\therefore$ (5, 0) is a solution.

$\therefore$ (−3, 4), (2, 1.5), (3, 1) and (5, 0) are four sets of solutions required.

Additional Example 11.2

Example 11.2 Setting up and solving a linear equation in two unknowns

In a bag of 3 600 g of oranges and apples, each orange weighs 300 g and each apple weighs 225 g.

(a) Let x and y be the number of oranges and apples respectively, set up an equation in x and y.

(b) According to the following table, find some possible number of oranges and apples.

x	3		9
y		8	

Solution

(a) Weight of all oranges = $300x$ g

Weight of all apples = $225y$ g

$\therefore$ $300x + 225y = 3\,600$

$4x + 3y = 48$ ◂ Divide both sides by 75 to simplify the equation.

(b) For the equation $4x + 3y = 48$,

substitute $x = 3$ into the equation,

$4(3) + 3y = 48$

$3y = 36$

$y = 12$

Substitute $y = 8$ into the equation,

$4x + 3(8) = 48$

$4x = 24$

$x = 6$

Substitute $x = 9$ into the equation,

$4(9) + 3y = 48$

$3y = 12$

$y = 4$

$\therefore$ Number of oranges and apples required:

3 oranges and 12 apples, 6 oranges and 8 apples, 9 oranges and 4 apples.

Classwork 11.2

Solution

A volunteer helper made Christmas gifts A and B for children. The time required for making a gift A and a gift B is 10 and 15 minutes respectively. The volunteer helper spent 120 minutes in making the gifts.

(a) Let x and y be the number of gifts A and B made respectively, set up an equation in x and y.

(b) According to the following table, find some possible number of gifts A and B.

x	3		9
y		4	

11.2 Graphs of Linear Equations in Two Unknowns

Class Activity 11.2

Aim: To investigate the graphs of linear equations in two unknowns

Show Ans.

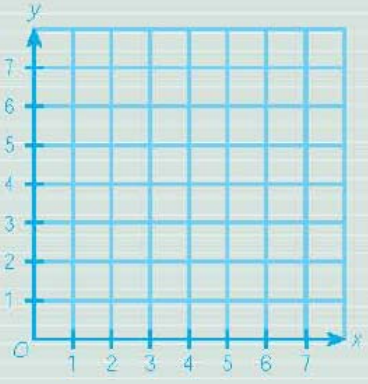
(a) For the equation $y = x + 1$, complete the following table.

x	1	2	3	4	5
y	2				

(b) Use ordered pairs to rewrite the results of **(a)**. [The first one is done as an example.]

(1, 2), (_____, _____), (_____, _____), (_____, _____), (_____, _____)

(c) Mark the points of the ordered pairs in **(b)** on the given rectangular coordinate plane. Join all points.



(d) What kind of graph do you get?

_____

Student's CD I.T. Tool

From the above Class Activity, we know that

> After marking all solutions of a linear equation in two unknowns on a rectangular coordinate plane, a straight line can be obtained. This straight line is called the **graph** of a linear equation in two unknowns.

graph 圖像

e.g. Consider the equation $2x+3y=18$.

x	0	3	9
y	6	4	0

The graph of the equation $2x+3y=18$ is as follows:

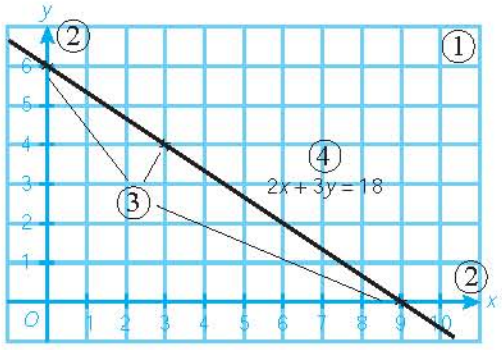


Figure 11.1

We should note the following when drawing a graph of linear equation in two unknowns.

① Choose a rectangular coordinate plane with a suitable scale.

② Label each axis with suitable markings.

③ Mark at least three points to ensure a correct straight line is drawn.

④ Label the equation of the straight line.

From the above, we can see that the name 'linear equation in two unknowns' comes from the fact that its graph is a straight line.

Additional Example 11.3

Example 11.3 Drawing a graph of an equation

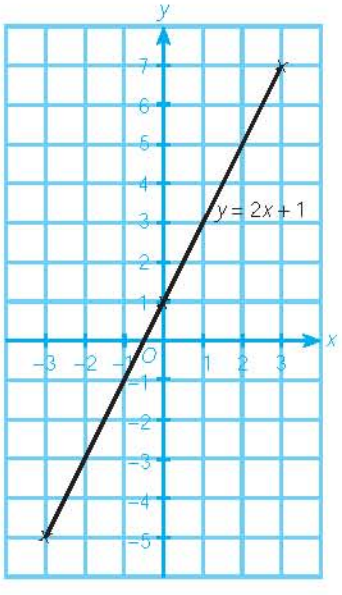
Draw the graph of the equation $y=2x+1$ from $x=-3$ to $x=3$ on a rectangular coordinate plane.

Solution

[Analysis: Mark three points on a rectangular coordinate plane. Then join all the points by a straight line. The straight line is the graph of $y=2x+1$.]

$y=2x+1$

x	-3	0	3
y	-5	1	7

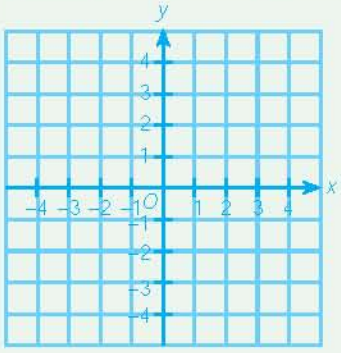


Classwork 11.3

Solution

Draw the graph of the equation $y=x-1$ from $x=-3$ to $x=3$ on a rectangular coordinate plane.

x	-3	0	3
y			



Additional Example 11.4

Example 11.4 Drawing a graph of an equation

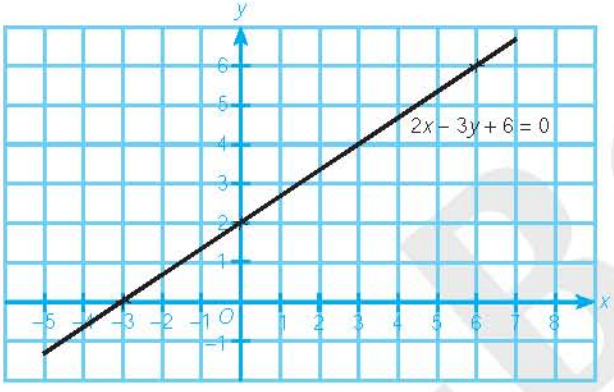
Draw the graph of the equation $2x-3y+6=0$ from $x=-5$ to $x=7$ on a rectangular coordinate plane.

Solution

[Analysis: Through a few trials of substituting integral values of x into the equation, we can obtain some integral values of x and y that satisfy the equation. Use these values to mark the points of the graph on the rectangular coordinate plane.]

$2x-3y+6=0$

x	-3	0	6
y	0	2	6



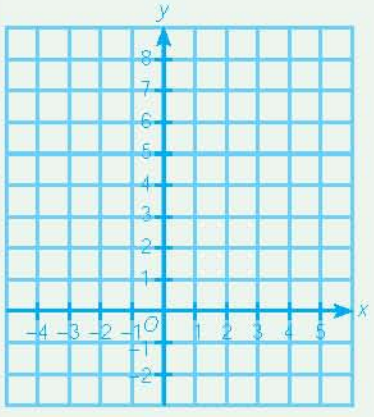
Note: If the marked points lie on the intersection of the grid, a more accurate graph can be obtained. Therefore, we usually choose points with both their x- and y-coordinates in integral values for drawing.

Classwork 11.4

Solution

Draw the graph of the equation $3x+2y=7$ from $x=-3$ to $x=4$ on a rectangular coordinate plane.

x			
y			



Skills Upgrading Corner 11.1

Solution

1. In a store, the prices of each pack of soya bean milk and green tea are \$2.4 and \$4 respectively. It is known that x packs of soya bean milk and y packs of green tea were sold one day and the total income was \$100.

(a) Set up an equation in x and y.

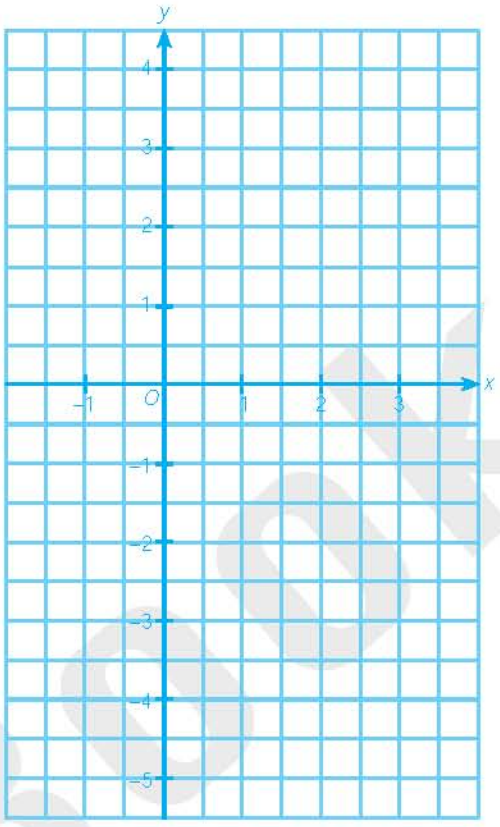
(b) In the following table, find the possible number of packs of soya bean milk and green tea sold.

x	5		30	
y		10		4

Solution

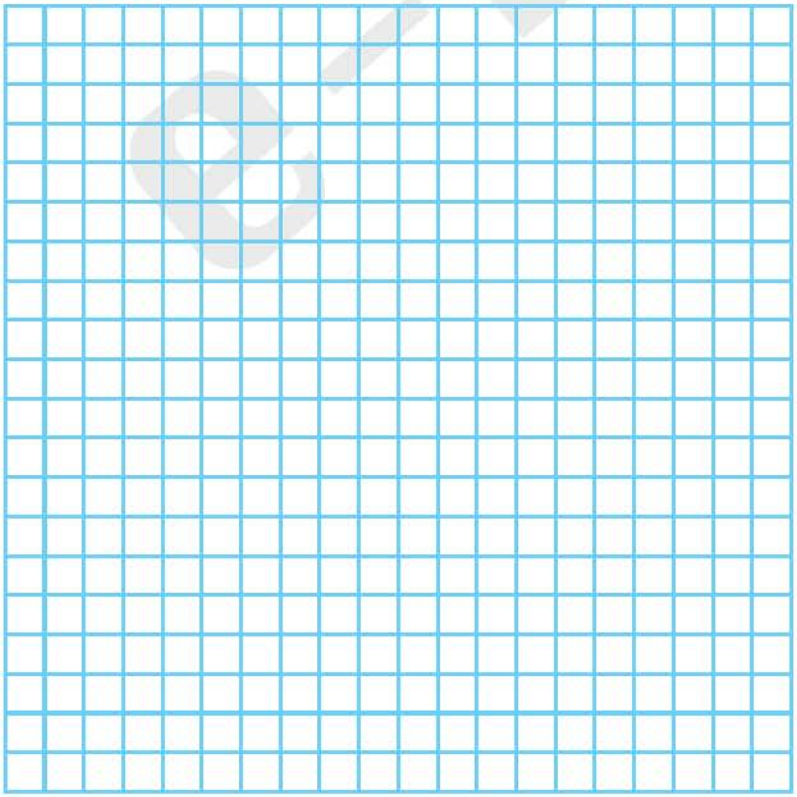
2. Draw the graph of the equation $4x-2y=5$ from $x=-1$ to $x=3$ on the given rectangular coordinate plane.

x	-1	0	3
y			



Solution

3. Draw the graph of the equation $3x-4y-9=0$ from $x=-2$ to $x=6$ on the rectangular coordinate plane.



Exercise 11A

[Graph paper is provided in the Appendix.]

Level 1

1. Complete the tables for the given equations.

(a) $y = 6 - 3x$

x	-3	0	1	3
y				

(b) $x = 9 - 3y$

x				
y	-4	-2	-1	1

(c) $x + y = 7$

x	-2	2	4	6
y				

2. For each of the following tables, find four of the solutions of the given equation in the form of ordered pairs.

(a) $x + 3y = 11$

x	-4		8	
y		3		-1

(b) $2x + y = 10$

x		6		3.5
y	-3		1	

(c) $3x - y = 4$

x	-1		1	
y		-4		2

3. Complete the table and draw the graph of each of the following equations on a rectangular coordinate plane.

(a) $y = x + 3$

x	-4	0	2
y			

(b) $y = 2(x + 1)$

x	-3	1	3
y			

(c) $y = \frac{1}{2}(6 - x)$

x	-4	0	8
y			

4. Draw the graph of the equation $y = 2x - 3$ from $x = -3$ to $x = 4$ on a rectangular coordinate plane.

Level 2

5. Draw the graph of the equation $x + y = 5$ from $x = -3$ to $x = 4$ on a rectangular coordinate plane.

6. Draw the graph of the equation $x - 3y = 4$ from $x = -4$ to $x = 5$ on a rectangular coordinate plane.

7. Draw the graph of the equation $4x + 3y + 20 = 0$ from $x = -3$ to $x = 5$ on a rectangular coordinate plane.

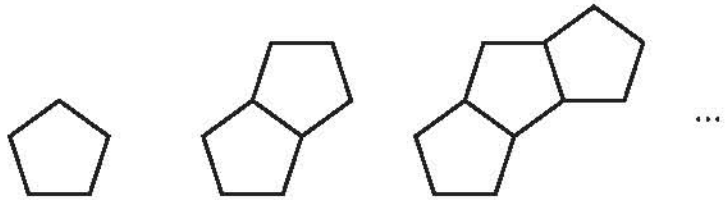
8. Calvin has bought some \$1.4 and \$1.8 stamps. Let x and y be the quantities of \$1.4 stamps and \$1.8 stamps bought respectively.

(a) Set up an equation in x and y if the total cost of the stamps is \$32.

(b) According to the following table, find some possible quantities of stamps of each kind bought.

x	1		
y		10	3

9. The following is a sequence of figures formed by toothpicks.



The 1st figure The 2nd figure The 3rd figure

By observation,
number of toothpicks in the 1st figure $= 4(1)+1$,
number of toothpicks in the 2nd figure $= 4(2)+1$,
number of toothpicks in the 3rd figure $= 4(3)+1$.

(a) Find the respective number of toothpicks in the 4th and 5th figures.

(b) Write an equation in x and y where y represents the number of toothpicks in the xth figure.

(c) Draw the graph of the equation obtained in **(b)** from $x = 1$ to $x = 5$ on a rectangular coordinate plane.

11.3 Points on the Graphs of Linear Equations in Two Unknowns

Class Activity 11.3

Aim: To explore the relationship between solutions of a linear equation in two unknowns and the points on its graph

Figure I shows the graph of the equation $x - y = 1$.

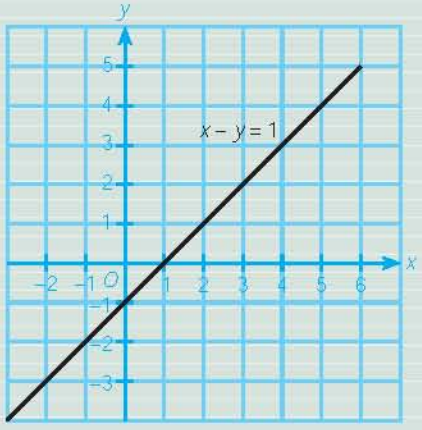


Figure I

1. (a) Choose any three points which lie on the graph of $x - y = 1$, and write down their coordinates.

Show Ans.

(_____, _____), (_____, _____), (_____, _____)

(b) Substitute the values of the above coordinates into the left-hand side of the equation $x - y = 1$ to check whether it is equal to the right-hand side.

For (_____, _____), L.H.S. = _____

For (_____, _____), L.H.S. = _____

For (_____, _____), L.H.S. = _____

(c) Are the above three points solutions of the equation $x - y = 1$?

☐ Yes ☐ No

2. (a) Complete the following table with four of the solutions of the equation $x - y = 1$.

Show Ans.

x	1		3	
y		1		5

(b) Express the results of **(a)** in the form of ordered pairs and mark these points on Figure I.

_____

(c) Do these points lie on the graph of the equation $x - y = 1$?

☐ Yes ☐ No

3. (a) Choose any three points which do **NOT** lie on the graph of the equation $x - y = 1$, and write down their coordinates.

Show Ans.

(_____, _____), (_____, _____), (_____, _____)

(b) Substitute the values of the above coordinates into the left-hand side of the equation $x - y = 1$ to check whether it is equal to the right-hand side.

For (_____, _____), L.H.S. = _____

For (_____, _____), L.H.S. = _____

For (_____, _____), L.H.S. = _____

(c) Are the above three points solutions of the equation $x - y = 1$?

☐ Yes ☐ No

From the Class Activity, we can see that

> The coordinates of all the points on the graph of a linear equation in two unknowns are solutions of the equation.
>
> Conversely, all ordered pairs which satisfy the equation are coordinates of points lying on the graph of the equation.

Substituting a solution of an equation back to the equation can surely balance both sides of the equation. We say that the ordered pair satisfies the equation.

Each point on the graph of a linear equation in two unknowns represents a solution of the equation. Since the graph is a straight line and there are infinitely many points on a line, a linear equation in two unknowns has an infinite number of solutions.

Additional Example 11.5

Example 11.5 Checking whether a given point is on the graph of an equation or not

Does $A(4, 3)$ lie on the graph of the equation $y = x + 1$?

Solution

[Analysis: Since the ordered pair of a point lying on the graph must satisfy the equation, the ordered pair (4, 3) is substituted into the equation for checking.]

Substitute $x = 4$ and $y = 3$ into the equation,

$$\text{L.H.S.} = y = 3$$

$$\text{R.H.S.} = x + 1 = 4 + 1 = 5$$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ (4, 3) does not satisfy the equation.

i.e. $A(4, 3)$ does not lie on the graph of the equation $y = x + 1$.

Classwork 11.5 Solution

Do the following points lie on the graph of the equation $y = 3x - 2$?

(a) $A(1, 1)$

(b) $B(2, 2)$

(c) $C(-2, -1)$

Additional Example 11.6

Example 11.6 Getting information from the graph of an equation

(a) Draw the graph of the equation $y = 1 - 2x$ from $x = -3$ to $x = 2$ on a rectangular coordinate plane.

(b) **(i)** Does (0, 0) lie on the graph?

(ii) Write down the coordinates of the points at which the graph cuts the x-axis and the y-axis.

(iii) From the graph, find the value of y when $x = -0.5$.

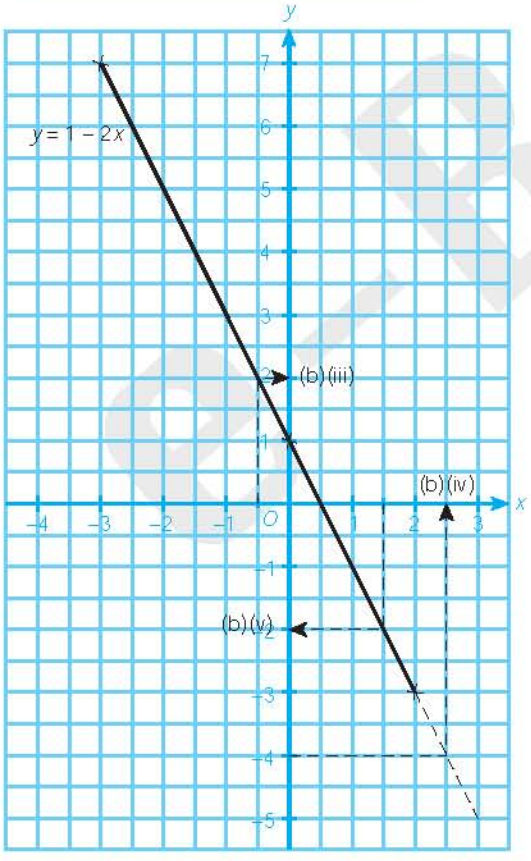
(iv) From the graph, find the value of x when $y = -4$.

(v) If $T(1.5, t)$ is a point lying on the graph, find the value of t.

Solution

(a) $y = 1 - 2x$

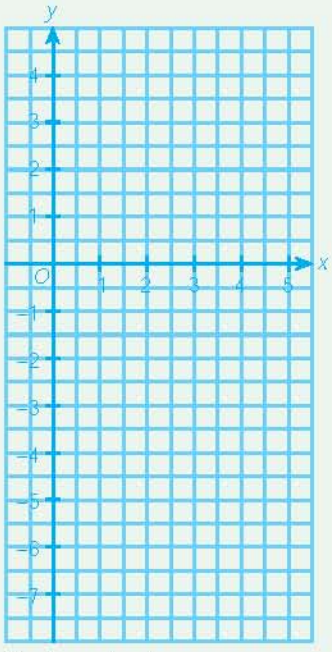
x	-3	0	2
y	7	1	-3



Classwork 11.6

Solution

(a) Draw the graph of the equation $y = 2x - 6$ from $x = 0$ to $x = 4$ on a rectangular coordinate plane.



(b) (i) Does (0, 0) lie on the graph?

(ii) Write down the coordinates of the points at which the graph cuts the x-axis and the y-axis.

(iii) From the graph, find the value of y when $x = 2$.

(iv) From the graph, find the value of x when $y = 4$.

(v) If $A(3.5, a)$ is a point lying on the graph, find the value of a.

(b) (i) From the figure,

$(0, 0)$ does not lie on the graph.

(ii) The graph cuts the x-axis at $(0.5, 0)$ and the y-axis at $(0, 1)$.

(iii) From the graph,

when $x = -0.5$, $y = 2$.

(iv) [Analysis: Extend the straight line, and then observe the corresponding value of x when $y = -4$.]

From the graph,

when $y = -4$, $x = 2.5$.

(v) From the graph,

when $x = 1.5$, $y = -2$.

$\therefore \quad t = -2$

Alternative method:

[Analysis: $\because$ $T(1.5, t)$ lies on the graph.
$\therefore$ $(1.5, t)$ satisfies the equation $y = 1 - 2x$.]

Substitute $x = 1.5$ and $y = t$ into the equation,

$t = 1 - 2(1.5)$
$= 1 - 3$
$= -2$

Additional Example 11.7

Example 11.7 Finding solutions by graphical method

Maria has got some 20¢ and some 50¢ coins in a total value of \$3.

(a) Let x and y be the number of 20¢ and 50¢ coins got by Maria respectively, set up an equation in x and y.

(b) Draw a suitable straight line on a rectangular coordinate plane and find all possible number of coins of each kind got by Maria.

Solution

(a) Amount of the 20¢ coins $= \$0.2x$
Amount of the 50¢ coins $= \$0.5y$

$\therefore \quad 0.2x + 0.5y = 3$

$2x + 5y = 30$

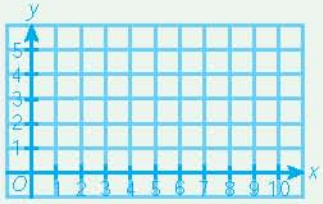
Classwork 11.7

Solution

The prices of small and large orange juice are \$10 and \$20 each pack respectively. Mike spent \$90 on orange juice.

(a) Let x and y be the number of packs of small and large orange juice bought by Mike respectively, set up an equation in x and y.

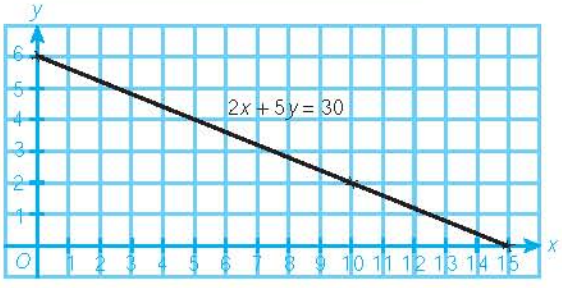
(b) Draw a suitable straight line on the given rectangular coordinate plane and find all possible number of packs of orange juice in each size bought.



(b) [Analysis: The required solutions can be found on the graph of $2x+5y=30$. Since all possible number of coins must be positive integers, we should choose the points where x and y are both positive integers from the graph.]

$2x+5y=30$

x	0	10	15
y	6	2	0



$\because$ The number of coins must be a positive integer.

$\therefore$ x and y must be positive integers.

According to the graph, only $x=5$, $y=4$ and $x=10$, $y=2$ satisfy the above condition.

$\therefore$ Maria has got five 20¢ coins and four 50¢ coins or ten 20¢ coins and two 50¢ coins.

Since it is given that Maria has got both 20¢ and 50¢ coins, $x=0$, $y=6$ and $x=15$, $y=0$ are not possible and should be excluded.

Note: We can also find the possible number of coins by trial and error, but this cannot ensure that all possible solutions are found. By graphical method, we can see all possible solutions easily.

Skills Upgrading Corner 11.2

Solution

1. Determine whether the following points lie on the graph of the equation $4x+5y=30$.

$A(5, 2)$, $B(-2, 10)$

Solution

2. Given the equation $x-6y+3=0$,

(a) **(i)** what is the y-coordinate of point A if the graph of the equation intersects the x-axis at A?

(ii) Hence, find the coordinates of A.

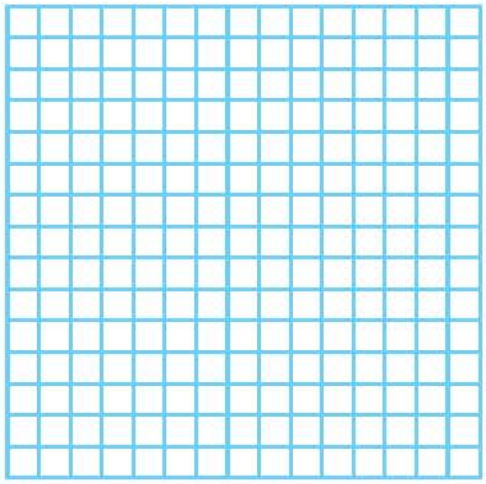
(b) Find the coordinates of the intersecting point of the graph of the equation and the y-axis.

Solution

3. Some candies are shared by 2 children equally and each of them gets x candies. However, if these candies are shared by 3 children equally, each of them can get y candies with 1 candy left.

(a) Set up an equation in x and y.

(b) It is known that x is a positive integer smaller than 12. Draw a suitable straight line on the rectangular coordinate plane and find all the possible values of x and y.



Exercise 11B

[Graph paper is provided in the Appendix.]

Level 1

1. Does the point representing $(3, -1)$ lie on the graph of the equation $3x - 2y = 7$?

2. Does $P(6, 5)$ lie on the graph of the equation $y = 2x - 3$?

3. Does $Q(2, 3)$ lie on the graph of the equation $2y = 5x - 4$?

4. Which of the following points do **NOT** lie on the graph of the equation $y = 3x + 1$?

$A(4, 10)$, $B(-1, -2)$, $C(0, 0)$, $D(0.5, -0.5)$

5. Which of the following points lie on the graph of the equation $y = \frac{x}{2} + 2$?

$P(0, 2)$, $Q(-1, -2)$, $R(3, 3\frac{1}{2})$, $S(-2, 1)$

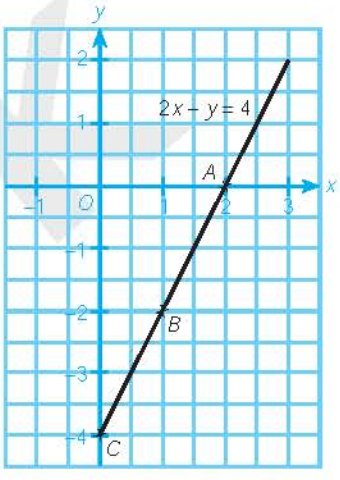
6. If $K(-2, k)$ lies on the graph of the equation $y = \frac{1}{2}x + 5$, find the value of k.

7. If $H(h, 3)$ lies on the graph of the equation $y = 4x - 5$, find the value of h.

8. If the graph of the equation $8x + 5y - 40 = 0$ cuts the x-axis and y-axis at points A and B respectively, find the coordinates of A and B.

Level 2

9. The figure shows the graph of the equation $2x - y = 4$ from $x = 0$ to $x = 3$.

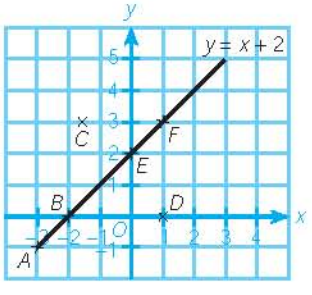


   (a) Find the coordinates of A, B and C.

   (b) If $D(3, d)$, $E(0.5, e)$ and $F(f, 1)$ lie on the graph of $2x - y = 4$, find the values of d, e and f.

   (c) Does $G(5, 6)$ lie on the graph of the equation $2x - y = 4$?

10. The figure shows the graph of the equation $y = x + 2$ from $x = -3$ to $x = 3$.



   (a) For points A to F,

   (i) which points lie on the graph of the equation $y = x + 2$?

   (ii) which points do not lie on the graph of the equation $y = x + 2$?

   (b) Are the following ordered pairs solutions of the equation $y = x + 2$?

   (i) $(0, 0)$ **(ii)** $(2, -2)$

   (iii) $(-1, 1)$ **(iv)** $(-4, -2)$

   (v) $(4, 6)$ **(vi)** $(9, 11)$

   (c) Write down the coordinates of the points at which the graph cuts the x-axis and the y-axis.

11. **(a)** Draw the graph of the equation $3 - y = x$ from $x = -3$ to $x = 5$ on a rectangular coordinate plane.

   (b) **(i)** Does $(0, 0)$ lie on the graph?

   (ii) Write down the coordinates of the points at which the graph cuts the x-axis and the y-axis.

   (iii) From the graph, find the value of y when $x = -1.5$.

   (iv) From the graph, find the value of x when $y = -1.5$.

   (v) If $P(p, 0.5)$ is a point lying on the graph, find the value of p.

12. **(a)** Draw the graph of the equation $y = 3x - 6$ from $x = -2$ to $x = 6$ on a rectangular coordinate plane.

(b) Mark the points $A(2, -8)$, $B(4, 6)$, $C(0, -6)$ and $D(3, 0)$ on the graph.

(c) Which of the points A, B, C and D lie on the graph?

(d) If $P(p, 3)$ and $Q(-1.5, q)$ are points on the graph, find the values of p and q.

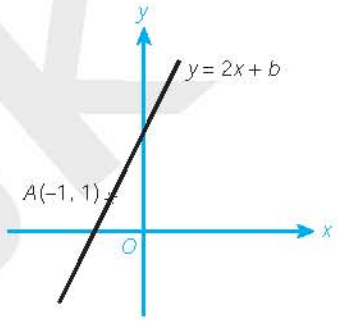
13. **(a)** If $A(-3, 0)$ is a point on the graph of the equation $2x - 3y = h$, find the value of h.

(b) From the result of **(a)**, draw the graph of the equation $2x - 3y = h$ from $x = -6$ to $x = 6$ on a rectangular coordinate plane.

14. In the figure, $A(-1, 1)$ is a point on the graph of the equation $y = 2x + b$.

(a) Find the value of b.

(b) Find the coordinates of the points at which the graph cuts the x-axis and the y-axis.



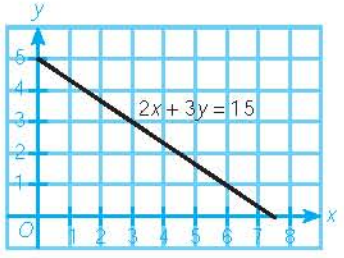
15. **(a)** Which quadrant does the graph of the equation $5x - 2y + 40 = 0$ never pass through?

(b) On each quadrant, write down the coordinates of a point on the graph of the equation $5x - 2y + 40 = 0$.

16. Each bean pudding costs \$2 and each egg tart costs \$3. Judy has spent \$15 on some bean puddings and some egg tarts.

(a) Let x and y be the quantities of bean puddings and egg tarts bought respectively, set up an equation in x and y.

(b) Use the figure on the right to find all possible quantities of bean puddings and egg tarts bought.



17. *Ballpoint pens* in brand A and brand B were sold at \$6 and \$4 each respectively. Elaine spent a total of \$36 on ballpoint pens.

(a) Let x and y be the number of brand A and brand B ballpoint pens bought respectively, set up an equation in x and y.

(b) Draw a suitable straight line on a rectangular coordinate plane and find all possible number of ballpoint pens in each brand bought by Elaine.

ballpoint pen 原子筆

18. Sandra has got some \$20 and some \$50 banknotes with a total value of \$400.

(a) Let x and y be the number of \$20 and \$50 banknotes she has got respectively, set up an equation in x and y.

(b) **(i)** Draw a suitable straight line on a rectangular coordinate plane and find all possible number of banknotes of each kind she has got.

(ii) If the number of \$50 banknotes is more than that of \$20 banknotes, find the number of banknotes of each kind.

Chapter Summary

Chapter Summary

Fact to Remember

1. If an equation has two unknowns, and the highest degree of all terms in the equation is 1, it is called a linear equation in two unknowns, e.g. $y = 2x + 1$, $5x + 3y = 2$.

2. A line joining the points of which their coordinates are solutions of an equation is called the graph of the equation.

3. The coordinates of all the points on the graph of an equation are solutions of the equation.

4. All ordered pairs which satisfy the equation are coordinates of points lying on the graph of the equation.

Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. The entrance fees of a museum are \$18 per adult and \$10 per child. The total entrance fee for x adults and y children is \$190. 11.1

(a) Set up an equation in x and y.

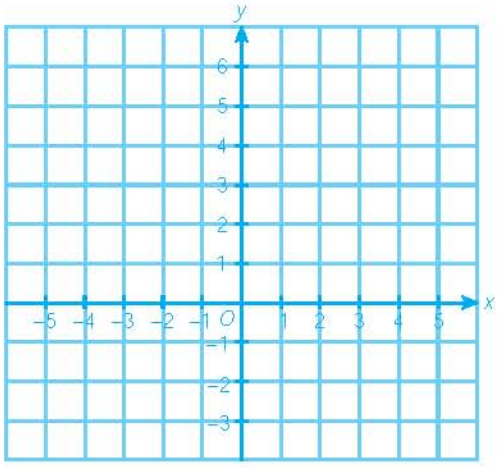
The required equation is _____.

(b) If there are 5 adults, find the number of children there.

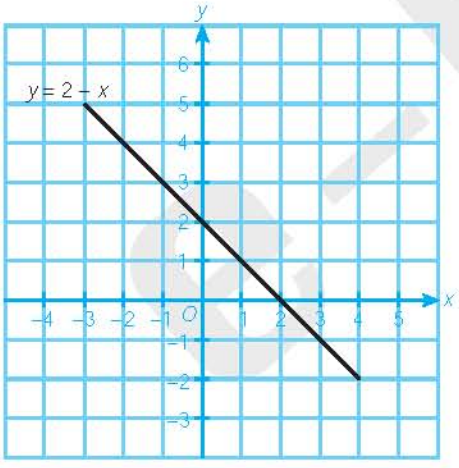
2. **(a)** Complete the following table for the equation $y = x + 2$. 11.2

x	-3	0	3
y			

(b) Draw the graph of the equation $y = x + 2$ from $x = -3$ to $x = 3$ on the given rectangular coordinate plane.



3. The figure shows the graph of the equation $y = 2 - x$ from $x = -3$ to $x = 4$. 11.3



(a) $A(-3, 4)$ lies / does not lie on the graph of the equation $y = 2 - x$.

(b) If the graph of the equation $y = 2 - x$ cuts the y-axis at $B(0, b)$, find the value of b.

Revision Exercise 11

Level 1

1. Complete the following tables for the given equations.

(a) $y = 3x + 1$

x	-3	-1	1	3
y				

(b) $y = 1 - \frac{x}{3}$

x	-12	-6	0	6
y				

(c) $9x - 2y - 15 = 0$

x	-1		2	
y		-7.5		6

2. Which of the following points lie on the graph of the equation $2x + y = 6$?
$A(-1, 7)$, $B(0, 6)$, $C(2, 5)$, $D(3, 0)$

3. Which of the following ordered pairs satisfy the equation $2y = 3x + 1$?
$(0, 0)$, $(1, 2)$, $(\frac{2}{3}, 1\frac{1}{2})$, $(-\frac{1}{2}, -\frac{1}{2})$

4. **(a)** Complete the following table for the equation $y = x + 4$.

x	-4	0	3
y			

(b) Draw the graph of the equation $y = x + 4$ from $x = -4$ to $x = 3$ on a rectangular coordinate plane.

5. **(a)** Complete the following table for the equation $y = \frac{x}{2} + 3$.

x	-6	0	6
y			

(b) Draw the graph of the equation $y = \frac{x}{2} + 3$ from $x = -6$ to $x = 6$ on a rectangular coordinate plane.

6. If the ordered pairs $(0, s)$ and $(t, -12)$ satisfy the equation $y = 3x - 6$, find the values of s and t.

7. If the ordered pairs $(m, 0)$ and $(6, n)$ satisfy the equation $3x - y = 27$, find the values of m and n.

8. If $H(h, -1)$ and $K(1, k)$ are points on the graph of the equation $y = 2x + 3$, find the values of h and k.

9. If $P(p, 0)$, $Q(0, q)$ and $R(-1, r)$ are points on the graph of the equation $x + 2y = 4$, find the values of p, q and r.

10. Write a linear equation in two unknowns, and find the coordinates of the points at which its graph cuts the x-axis and the y-axis.

11. If $(2, -4)$ is a solution of a linear equation in two unknowns, write down this equation and explain briefly.

Level 2

12. Draw the graph of each of the following equations on a rectangular coordinate plane.

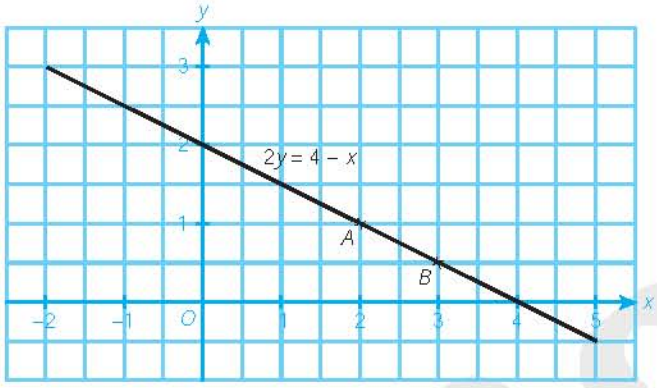
(a) $x + y + 2 = 0$ (from $x = -5$ to $x = 2$)

(b) $2y = x$ (from $x = -4$ to $x = 4$)

(c) $3x - 4y = 24$ (from $x = -4$ to $x = 4$)

(d) $0.2x + 0.5y - 1 = 0$ (from $x = -5$ to $x = 5$)

13. The figure shows the graph of the equation $2y = 4 - x$ from $x = -2$ to $x = 5$.



(a) Write down the coordinates of points A and B.

(b) Find the coordinates of the points at which the graph cuts the x-axis and the y-axis.

(c) If the graph is extended, will it pass through the following points?

(i) $P(6, -1.5)$

(ii) $Q(-5, 4.5)$

14. The figure shows the graph of the equation $3x - 2y = 5$ from $x = -3$ to $x = 5$.

(a) Write down the coordinates of points A and B.

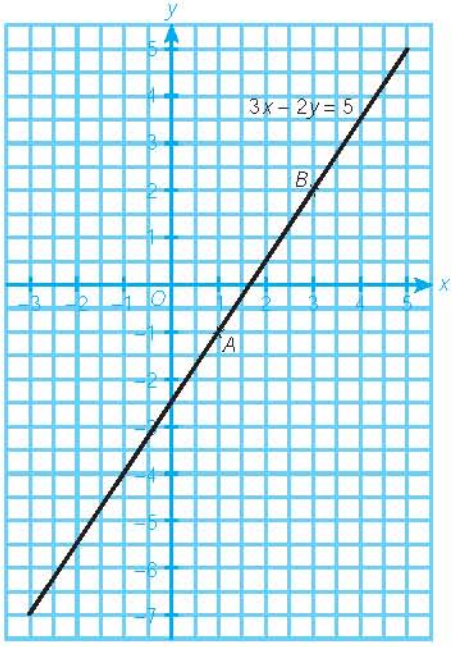
(b) If $C(-1, c)$ and $D(d, 3.5)$ are points on the graph, find the values of c and d.

(c) If $M(m, m)$ lies on the graph, find the coordinates of M.

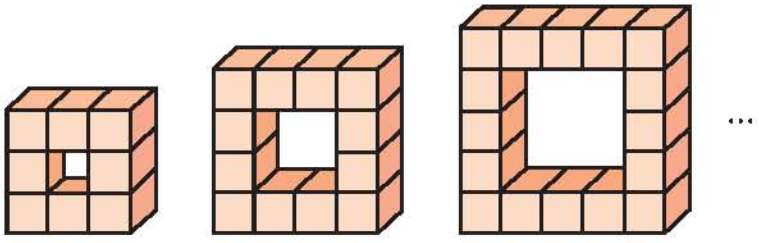
(d) If the graph is extended, will it pass through the following points?

(i) $P(-5, -10)$

(ii) $Q(10, 12)$



15. The following is a sequence of figures formed by cubes.



The 1st figure The 2nd figure The 3rd figure

By observation,

number of cubes in the 1st figure $= 4(1)+4$,
number of cubes in the 2nd figure $= 4(2)+4$,
number of cubes in the 3rd figure $= 4(3)+4$.

(a) Write an equation in x and y where y represents the number of cubes in the xth figure.

(b) Draw the graph of the equation in **(a)** from $x=1$ to $x=5$ on a rectangular coordinate plane.

(c) From the graph obtained in **(b)**, find the value of y when $x=10$.

16. **(a)** Draw the graph of the equation $y=-x+5$ from $x=-1$ to $x=6$ on a rectangular coordinate plane.

(b) Does $A(2.5, 2.5)$ lie on the graph?

(c) Find the coordinates of the points at which the graph cuts the x-axis and the y-axis.

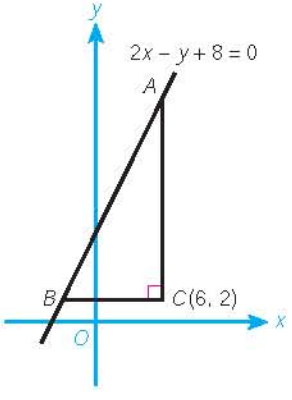
17. **(a)** Draw the graph of the equation $y=\frac{3}{2}x$ from $x=-4$ to $x=4$ on a rectangular coordinate plane.

(b) Find the coordinates of the points at which the graph cuts the x-axis and the y-axis.

(c) From the graph, find the value of x when $y=4.5$.

(d) Does the ordered pair $(24, 32)$ satisfy the equation $y=\frac{3}{2}x$?

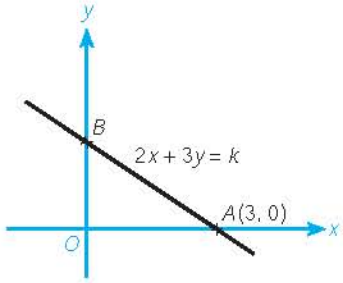
18. In the figure, ABC is a right-angled triangle, and the coordinates of C are $(6, 2)$. If points A and B lie on the graph of the equation $2x-y+8=0$,



(a) find the coordinates of A and B.

(b) From the result of **(a)**, find the area of ΔABC.

19. In the figure, the graph of the equation $2x + 3y = k$ passes through $A(3, 0)$ and cuts the y-axis at point B.



(a) Find the value of k.

(b) Find the coordinates of B.

(c) Find the area of ΔAOB.

20. Patrick has two sizes of boxes. The bigger one can hold 12 cans of soft drinks, and the smaller one can hold 8 cans. He is going to pack 72 cans of soft drinks.

(a) Let x and y be the number of bigger and smaller boxes Patrick has respectively, set up an equation in x and y.

(b) Draw a suitable straight line on a rectangular coordinate plane and find all possible number of boxes of each size Patrick has.

21. Extra Supermarket offered eggs in two different types of packing. There were 6 eggs in each carton of packing A and 4 eggs in each carton of packing B. It is known that Brian bought a total of 32 eggs in these two types of packing.

(a) Let x and y be the number of cartons of packing A and B Brian bought respectively, set up an equation in x and y.

(b) Draw a suitable straight line on a rectangular coordinate plane and find all possible number of cartons of each packing Brian bought.

22. In a Mathematics test, there were 2 sections with 10 questions each. Each question in Section A carried 4 marks, and each question in Section B carried 6 marks. It is known that John got 74 marks in the test.

(a) Let x and y be the number of correct answers John got in Section A and Section B respectively, set up an equation in x and y.

(b) It is known that John got 1 more correct answer in Section A than in Section B. Draw a suitable straight line on a rectangular coordinate plane and find the number of correct answers he has got in each section.

23. Students in S1A spent \$100 on paper for banners. The price of paper with patterns was \$10/m^2 and the price of coloured paper was \$5/m^2.

(a) Suppose they bought x m^2 of paper with patterns and y m^2 of coloured paper, set up an equation in x and y.

(b) **(i)** Draw the graph of the equation in **(a)**.

(ii) Could they possibly buy 8 m^2 of paper with patterns and 8 m^2 of coloured paper? Why or why not?

(iii) If the area of the coloured paper is twice that of the paper with patterns, find the area of the paper of each kind they bought.

24. It is known that the graph of the equation $ax - y + c = 0$ passes through $(0, -6)$.

(a) Find the value of c.

(b) If $P(3, 6)$ lies on the graph of the equation $ax - y + c = 0$, find the value of a.

(c) If $Q(q, q)$ lies on the graph of the equation $ax - y + c = 0$, find the coordinates of Q.

25. **(a)** Draw the graph of each of the following equations from $x = -2$ to $x = 5$ on a rectangular coordinate plane.

(i) $y = 2x - 4$

(ii) $y = 5 - x$

(b) Find the coordinates of the intersecting point of the graphs obtained in **(a)**.

(c) Does the ordered pair obtained in **(b)** satisfy both equations in **(a)**? Explain briefly.

MC Question

26. For the equation $2x + y - 8 = 0$, when $y = 4$, $x =$

A. 0.

B. 2.

C. 4.

D. 6.

When $x = 3$, $y =$

A. -5.

B. -1.

C. 2.

D. 7.

27. 3 kiwi fruits are \$2 cheaper than 4 pears. Suppose \$$x$ and \$$y$ are the prices of a pear and a kiwi fruit respectively, which of the following equations describes the relationship between x and y?

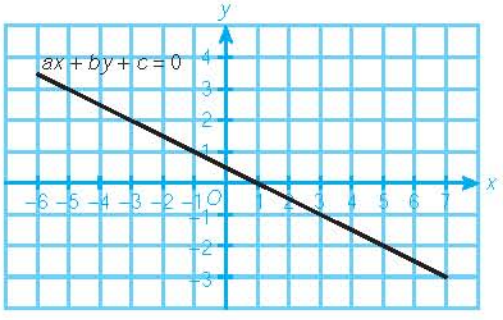
A. $4x - 3y = 2$

B. $4y - 3x = 2$

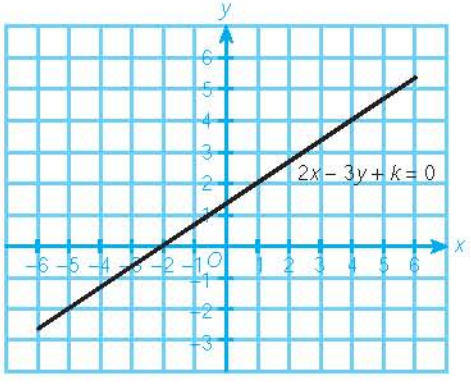
C. $3x - 4y = 2$

D. $3y - 4x = 2$

28. The figure is the graph of the equation $ax + by + c = 0$.



29. The figure is the graph of the equation $2x - 3y + k = 0$.



Which of the following points lies on the graph of the equation?

A. $A(-2, -5)$

B. $B(1, 2)$

C. $C(2, 0)$

D. $D(6, 6)$

30. Which of the following points lies on the graph of the equation $2x + 3y - 18 = 0$?

A. $A(-9, 12)$
B. $B(-3, -8)$
C. $C(6, 0)$
D. $D(9, 9)$

31. If $P(3, -4)$ lies on the graph of the equation $2x + 2y + k = 0$, $k =$

A. -2.
B. -1.
C. 1.
D. 2.

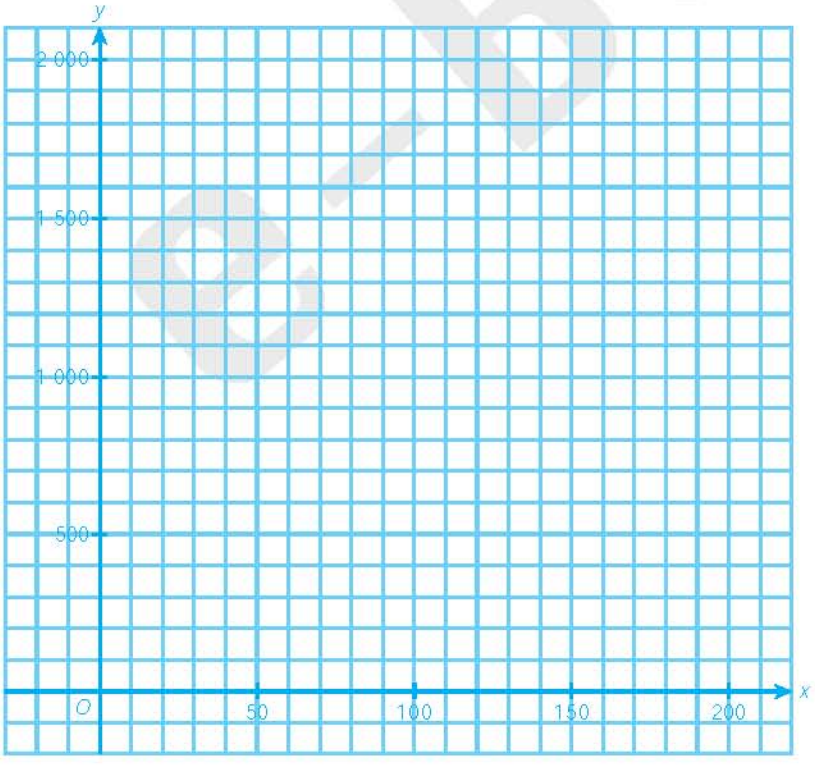
Problem-solving and Exploring

Hint for the Title Page Question

(a) Let $\$a$ be the basic fixed cost for printing comic books, $\$b$ be the variable cost of each copy printed and $\$y$ be the total cost for printing x copies of comic books. Write an equation in a, b, x and y.

(b) It is known that a and b are constants. Is the equation obtained in **(a)** a linear equation in two unknowns?

(c) It is known that it costs \$800 for printing 50 copies of comic books and \$2 000 for 200 copies. Mark the points (50, 800) and (200, 2 000) on the given rectangular coordinate plane.



(d) Draw a suitable straight line on the above rectangular coordinate plane and find the total cost for printing 100 copies of comic books.

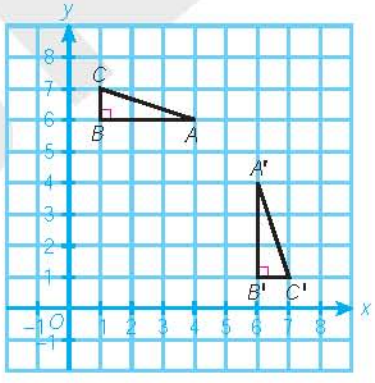
Additional Question

1. A hawker sold $\frac{2}{3}$ of all apples in the morning and ate one apple. In the afternoon, he sold $\frac{2}{3}$ of the remaining apples and then ate one again. In the evening, he sold $\frac{2}{3}$ of the apples left and then ate one more apple. There were still some apples left unsold at night.

(a) Let x and y be the number of the unsold apples and the original number of apples respectively, set up an equation in x and y.

(b) At least how many apples did the hawker have originally?

2. In the figure, ΔABC and $\Delta A'B'C'$ are symmetrical.



(a) Draw the axis of symmetry.

(b) Write down the coordinates of any three points on the axis of symmetry.

(c) Work out the equation of the axis of symmetry.